Nonlinear Dyn (2015) 79:1185–1194
DOI 10.1007/s11071-014-1735-0

ORIGINAL PAPER

Nonlinear control of triple inverted pendulum based on GA–PIDNN

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Received: 18 April 2014 / Accepted: 28 September 2014 / Published online: 10 October 2014
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Abstract The triple inverted pendulum is a nonlinear, dynamic and unsteady system. The traditional control methods of triple inverted pendulum have problems of limited control accuracy, slow responding. This kind of pendulum system is difficult to control due to the inherent instability, nonlinear behavior and difficultly in establishing a precise mathematical model. In addition, the back-propagation (BP) algorithm has the shortage of easy trapped in local minimum. The triple inverted pendulum control based on GA–PIDNN is proposed. The PID neural network (PIDNN) is a new kind of feedforward multi-layer network. Besides multi-layer forward networks traditional merit, such as approach the ability proceed together the calculation nonlinear transformation, its middle layer has the proportional (P) integral, (I) derivative, (D) dynamic characteristic. Genetic algorithm (GA) has good parallel design structure and characteristics of global optimization. The nonlinear identification model is established, and controller is designed via GA–PIDNN based on the combination of the merits GA and PIDNN in the research of triple inverted pendulum. In simulation, through the comparative study of GA–PIDNN and PIDNN optimized by BP (BP–PIDNN), simulations results show that GA is more accurate and effective.

Keywords Triple inverted pendulum · PIDNN · GA · The nonlinear identification model · Optimal control

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1 Introduction

As a typical model of multivariable, nonlinear, absolutely unsteady system, the triple inverted pendulum is usually used as a benchmark for verifying the effectiveness of a new control method, and research inverted pendulum system to theoretically have the profound meaning with methodology up [1,2].

In recent years, the "inverted pendulum" has been studied widely. Moreover, it is also a big theoretical topic in modern control research. For example, the adaptive control of rotary inverted pendulum system with time-varying uncertainties is realized [3], the stability and Hopf bifurcation in an inverted pendulum with delayed feedback control is finished [4], and the variable universe adaptive fuzzy controller based on variable gain H_∞ regulator ($VGH_\infty R$) is designed to stabilize a quadruple inverted pendulum [5]. The triple inverted pendulum system is difficult to establish a precise mathematical nonlinear model and realize nonlinear control due to the inherent instability, nonlinear behavior [6].

As is well known, conventional feedforward multilayer neural networks have many advantages so that

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they are most widely used in many fields of the industry. Neural network oriented genetic algorithm can overcome the shortcoming of slow convergent speed, immature convergence and lots of iterations [7].

But the general forward neural networks have no theoretical guidance which is used to determine the hidden-layer neurons' number and mode of connection. Besides, most of conventional feedforward multi-layer neural networks are static networks. They cannot easily deal with dynamic information and often need to add additional links when realize the accurate identification and control of dynamic systems [8].

PIDNN is a new kind of neural network [9]. It integrates the advantages of both PID control and neural structure. Because of PIDNN's dynamic characteristics, it has a better performance than general forward network in processing dynamic information. PIDNN has a fixed structure of 2-3-1, which can be obtained with corresponding theories. The weights of network can be adjusted by the control effect.

GA is a kind of directly search method which can be used to global search based on the mechanics of natural selection and natural genetics [10]. Besides, GA has the characteristics of global optimization and effectively prevent converging into local optimal solution which does not rely on the specific problems [11]. It does not depend on the gradient information too.

In addition, BP has some defects such as slow convergence rate and getting into local minimum. To sum up, the nonlinear identification model and controller of triple inverted pendulum are designed based on GA–PIDNN by the combination of the merits GA and PIDNN.

The rest of the paper is organized as follows. Section 2 presents the optimization of PIDNN by GA. Section 3 specifies the design of nonlinear identification model based on GA–PIDNN. Section 4 is the design of control system based on GA–PIDNN. System simulation results are introduced in Sect. 5, including the performance behavior comparing between GA–PIDNN and BP–PIDNN. Finally, the conclusion is given in Sect. 6.

2 Optimized PIDNN by GA

The core idea of the PIDNN is to incorporate proportional, integral and derivative functions into hidden layer of the neural network. It has the capability of directly controlling the complex control systems.

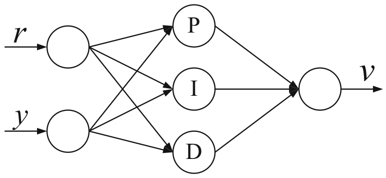

Fig. 1 Structure of SPIDNN

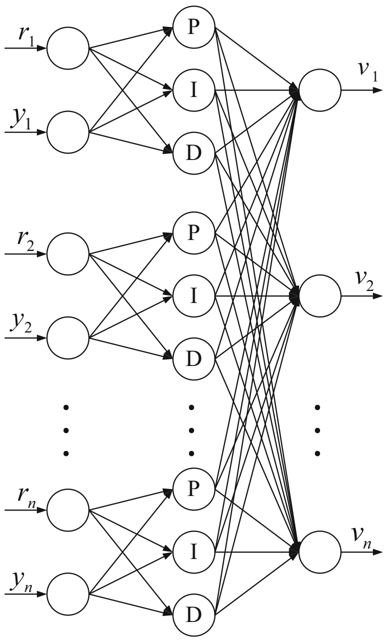

Fig. 2 Structure of MPIDNN

2.1 Structure and principle of PIDNN

PIDNN is a dynamic network that exists single-output PIDNN (SPIDNN) and multi-output PIDNN (MPIDNN). The basic structure of SPIDNN is 2-3-1, and the structure of SPIDNN is shown in Fig. 1.

SPIDNNs are composed of input layer, hidden layer and output layer. The input layer has two neurons, the hidden layer has three and the output layer has only one.

The input layer has two proportional (P) neurons, one receives system setting input and another connects system output. The hidden layer has three neurons, which are proportional (P) neuron, integral (I) neuron and derivative (D) neuron. The output layer has only one neuron which completes the control output duty [11]. MPIDNN needs to combine several SPIDNN in parallel. The structure of MPIDNN is shown in Fig. 2.

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2.2 Algorithm of PIDNN

The input layer of MPIDNN has $2n$ neurons, and the input-output functions are as follows

$$
\begin{aligned}
net_{si}(k) &= r_s(k) \\
net_{si}(k) &= y_s(k)
\end{aligned}
\tag{1}
$$

where s is the number of subnet ($s = 1, 2, \ldots, n$), i is the number of subnet's input layer ($i = 1, 2$). k is computer sample time.

The states of input-layer neurons are

$$u_{si}(k) = net_{si}(k) \tag{2}$$

The input-layer neurons are linear neurons [12]. Because of the linear relationship between input and output, the outputs of input layer neurons can be expressed as follows

$$x_{si}(k) = u_{si}(k) \tag{3}$$

The hidden layer includes n proportional (P) neurons, n integral (I) neurons and n derivative (D) neurons. The inputs are

$$net'_{sj}(k) = \sum_{i=1}^{2} w_{sij} x_{si}(k) \tag{4}$$

where j is the index of hidden-layer neurons ($j = 1, 2, 3$), $x_{si}(k)$ is the output of input-layer neurons and w_{sij} is the weight from input layer to hidden layer.

There are three state functions of the hidden-layer neurons. The state of P-neuron is

$$u'_{s1}(k) = net'_{s1}(k) \tag{5}$$

The state of I-neuron is

$$u'_{s2}(k) = u'_{s2}(k-1) + net'_{s2}(k) \tag{6}$$

The state of D-neuron is

$$u'_{s3}(k) = net'_{s3}(k) - net'_{s3}(k-1) \tag{7}$$

The outputs of hidden-layer neurons are

$$
x'_{sj}(k) = \begin{cases} 1, & u'_{sj}(k) > 1 \\ u'_{sj}(k), & -1 \le u'_{sj}(k) \le +1 \\ -1, & u'_{sj}(k) < -1 \end{cases}
\tag{8}
$$

The output layer has n neurons, the inputs of the neurons are

$$net''_h(k) = \sum_{s=1}^{n} \sum_{j=1}^{3} w'_{sjh} x'_{sj}(k) \tag{9}$$

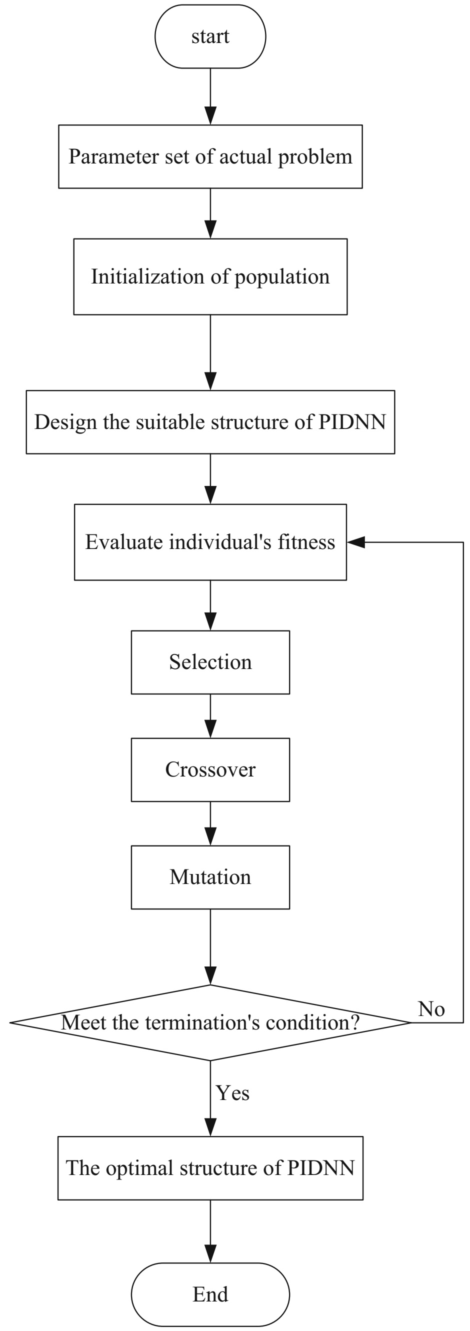

Fig. 3 Flow diagram of PIDNN optimized by GA

where h is the index of output-layer neurons ($h = 1, 2, \ldots, n$), $x'_{sj}(k)$ is the output of hidden-layer neurons and w'_{sjh} is the weight from hidden layer to output layer.

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The states of output-layer neurons are

$$u''_h(k) = net''_h(k) \quad (10)$$

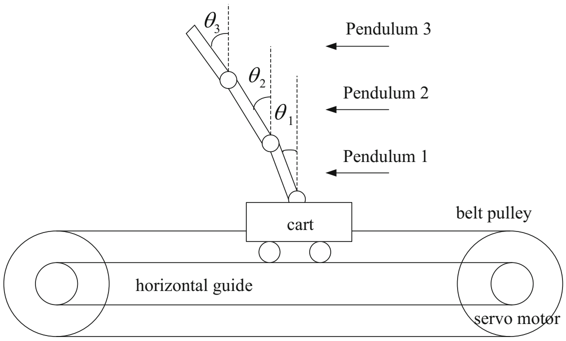

Fig. 4 Structure of the triple inverted pendulum

The outputs of output-layer neurons are

$$x''_h(k) = u''_h(k) \quad (11)$$

The input-output functions of output layer are the following proportional (P) functions

$$v_h(k) = x''_h(k) \quad (12)$$

The objective function of MPIDNN is

$$J = \sum_{p=1}^{n} E_p = \sum_{p=1}^{n} \sum_{k=1}^{l} [r_p(k) - y_p(k)]^2 \quad (13)$$

where p is the index of training samples ($p = 1, 2, \ldots, n$), l is the number of sampling points. $r_p(k)$ is the setting values of variables, $y_p(k)$ is the practical output values of variables. In the actual programming, the objective function is so.

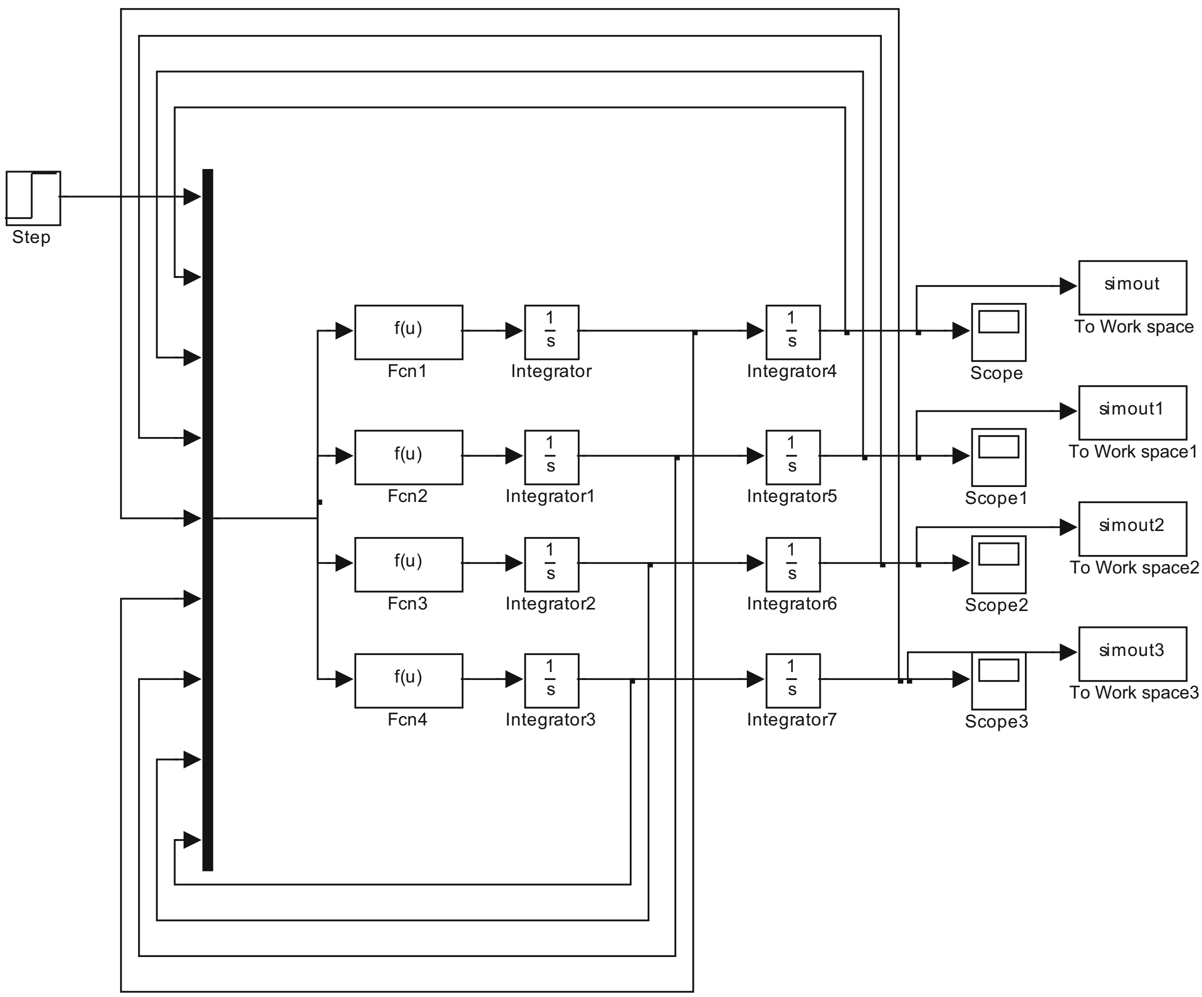

Fig. 5 Simulink model of the triple inverted pendulum

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2.3 Optimization process of GA–PIDNN

GA is a high efficient, parallel, global search method based on the mechanics of natural selection and natural genetics. It is used to deal with many individuals in the group at the same time, which ensures the performance of global search. The process of GA mainly includes selection, crossover and mutation operator. Selection operator is intended to improve the average quality of the population by the high-quality chromosomes, a better chance to get copied into the next generation [13].

In the process of crossover, two individuals are mated in order to exchange genetic information. The chromosomes, which represent the two paths, are broken at the same (random) place and combined together creating two new individuals [14]. The searching ability of GA is mainly entrusted by selection and crossover. The mutation operator is built into the proposed strategy to maintain population diversity and improve the ability of global exploration. Every point of the space can be searched because of the existence of mutation operator. GA can avoid falling into the local optimum in the process of searching and achieve a global optimal solution.

GA firstly encodes feasible solution into initial population. Then, the better region in the search space can be got by the actions of selection, crossover and mutation operations of any initial population. The cycle process will finally converge to the group individuals which are best adapted to the environment and obtain the optimal solution by decoding [15].

In the process of searching, GA only makes the fitness function as criteria and uses the individuals' values of fitness to search the optimal solution. Therefore, the fitness function affects the convergence rate and the problem whether to find the optimal path directly. The definition of the fitness function is critical [16]. This paper takes the objective function J of PIDNN as the fitness function and getting the minimum fitness function value is the optimization objective.

GA–PIDNN nonlinear identification model and controller are optimized by GA. To make the search process more convenient, GA toolbox and real number coding

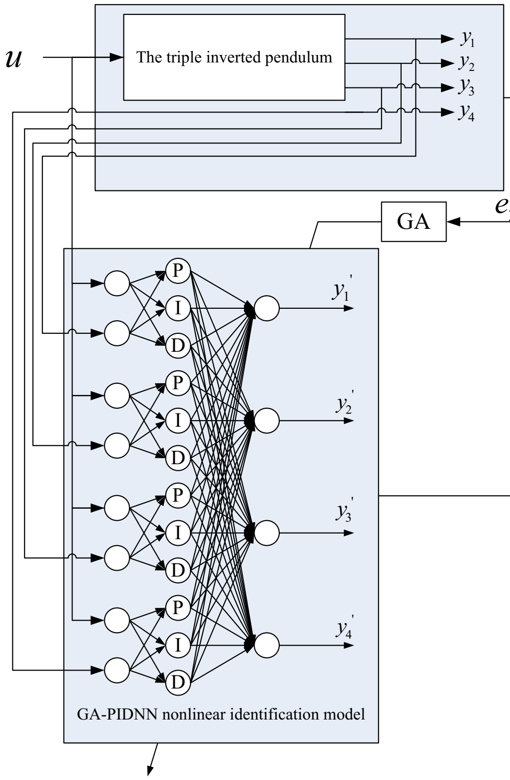

Fig. 6 Structure of GA–PIDNN nonlinear identification system

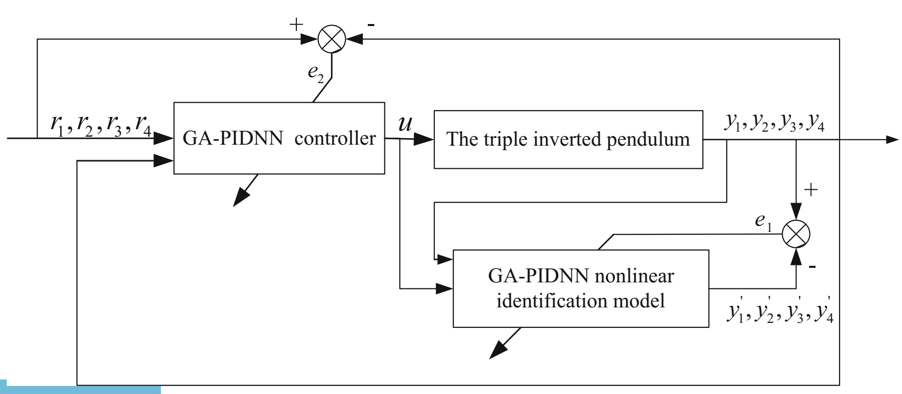

Fig. 7 Structure of the triple inverted pendulum control system

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method are used. The parameters of GA mainly include population size, stopping generations, crossover probability and mutation probability. In this paper, the default values of the parameters are used, population size is 20, crossover probability is 0.8, and mutation probability is adaptively ranges from 0.01 to 0.1. The flow diagram of PIDNN optimized by GA is shown in Fig. 3.

3 Design of nonlinear identification model based on GA–PIDNN

The triple inverted pendulum system consists of a cart placed on a track, pendulum 1, pendulum 2, pendulum 3, horizontal guide, servo motor and belt pulley [15]. Pendulum 1, pendulum 2 and pendulum 3 are connected by bearing, and the bearing can be rotated

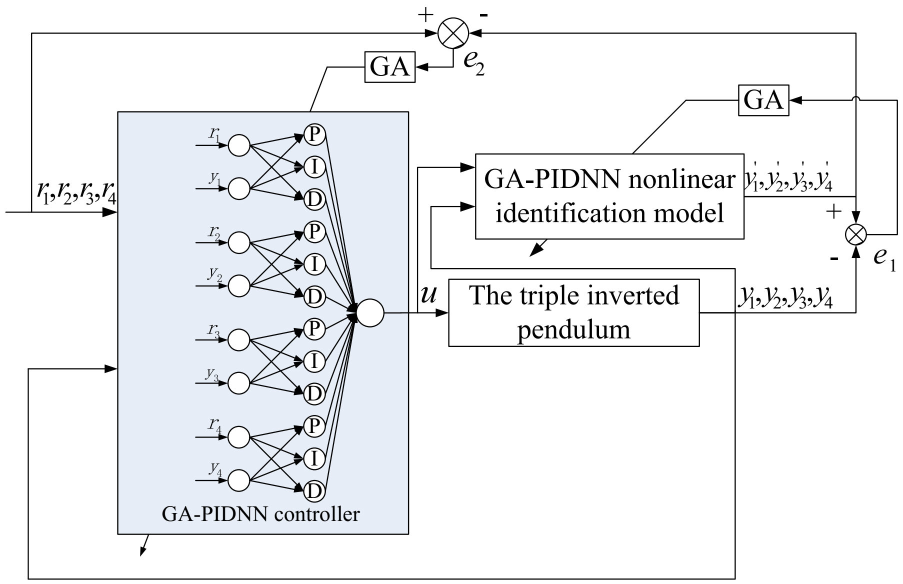

Fig. 8 Structure of GA–PIDNN control system

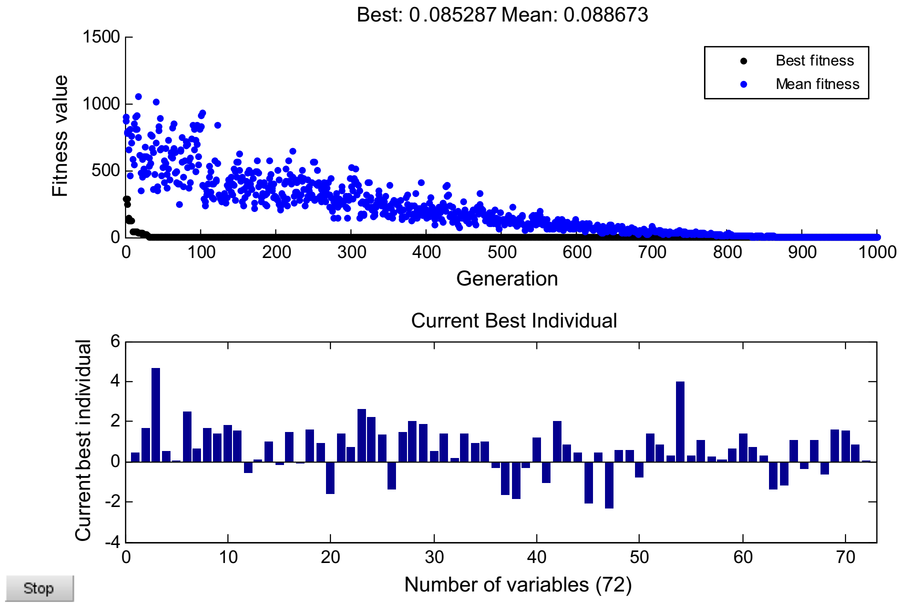

Fig. 9 GA optimization results of nonlinear identification system by GA

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freely in vertical plane of the parallel guide. The triple inverted pendulum device is shown in Fig. 4.

The multivariable system can be controlled by GA–PIDNN, and every controlled variable has perfect dynamic and static performances. GA–PIDNN can establish nonlinear identification model of controlled object based on the input/output data, even if the internal structure and parameter of actual system are unknown. The input/output data can be gained from actual system or anyone nonlinear mathematical model; to facilitate validating and simulating, it is obtained from a mathematical model which is shown as follows.

Based on the Lagrange equations of analytical mechanics, the inverted pendulum can be expressed as shown below:

$$W = F(X) \tag{14}$$

where W is $[\ddot{x}, \ddot{\theta}_1, \ddot{\theta}_2, \ddot{\theta}_3]$, and $x, \theta_1, \theta_2, \theta_3$ are values of displacement of cart, the angle of the pendulum 1, the angle of the pendulum 2 and the angle of the pendulum 3. X is the current state variables of system, $X = [x, \theta_1, \theta_2, \theta_3, \dot{x}, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]$. F is the nonlinear mapping function from X to W. The perfect, precise and nonlinear model is established with simulink based on formula 14, it is shown in Fig. 5.

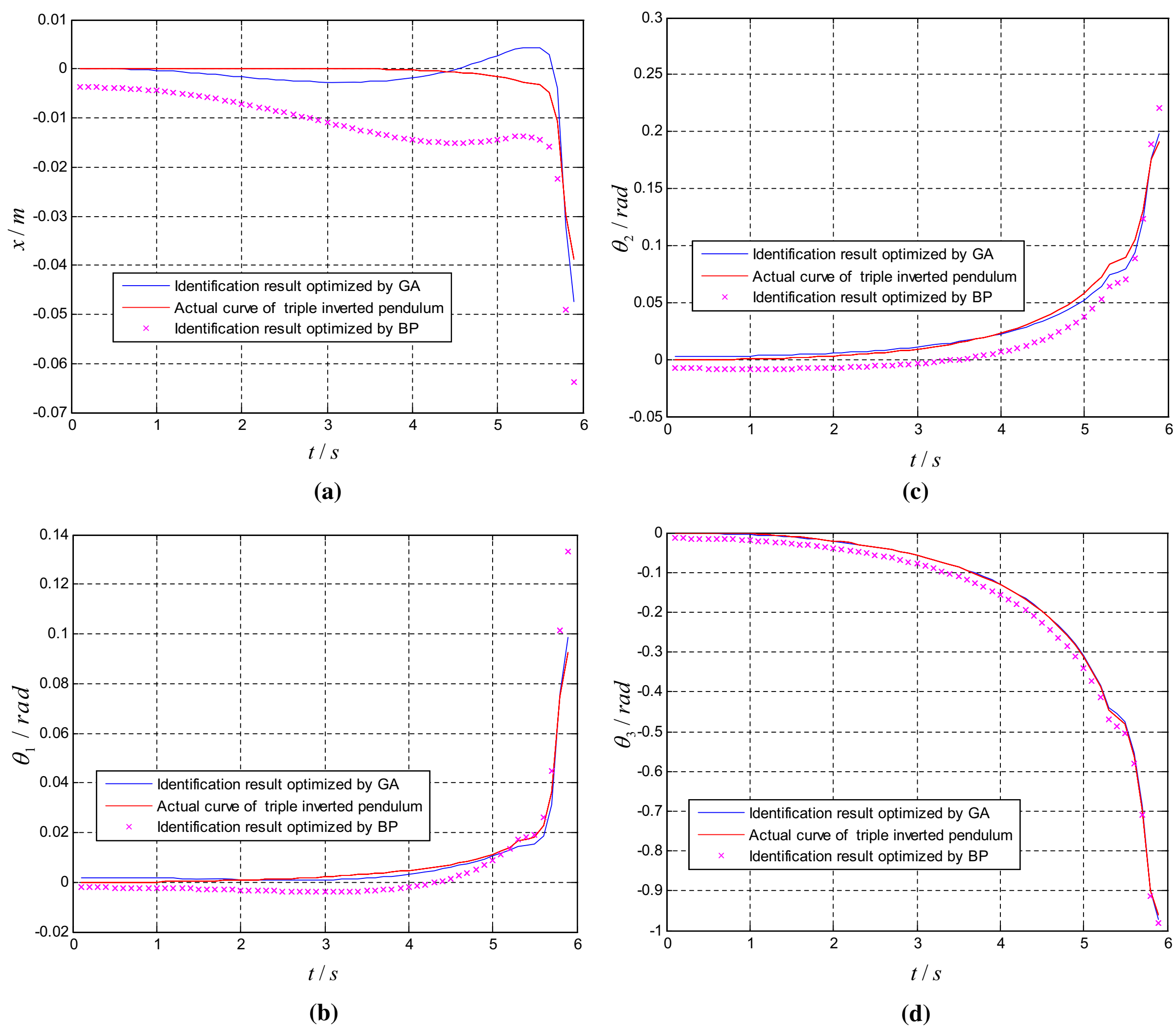

Fig. 10 The simulation results of nonlinear identification system. **a** The displacement of cart, **b** the angle of pendulum 1, **c** the angle of pendulum 2, **d** the angle of pendulum 3

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GA–PIDNN nonlinear identification model of triple inverted pendulum can achieve favorable tracking of complex nonlinear system when the actual system dynamics functions are unknown. The triple inverted pendulum has one input variable and four output variables; therefore, its GA–PIDNN identifier needs to combine four SPIDNNs in parallel, which contain 72 weights. The weights are composed of 24 weights from input layer to hidden layer and 48 weights from hidden layer to output layer. The structure of GA–PIDNN nonlinear identification system is shown in Fig. 6.

where r_1, r_2, r_3, r_4 are the setting values which are the displacement of cart, the angle of the pendulum 1, the angle of the pendulum 2 and the angle of the pendulum 3. y_1, y_2, y_3, y_4 are the measurements of the corresponding control variables and y_1', y_2', y_3', y_4' are the corresponding outputs of GA–PIDNN nonlinear identification model. Besides, u is the external force which controls the cart's movement.

4 Design of control system based on GA–PIDNN

The GA–PIDNN controller is designed for the triple inverted pendulum. PIDNN's weights are adjusted by GA, and it can achieve perfect performances by fast learning. The structure of triple inverted pendulum control system is shown in Fig. 7.

With triple inverted pendulum, its GA–PIDNN controller needs to combine four SPIDNN in parallel, which contain 36 weights. The weights are composed of 24 weights from input layer to hidden layer and 12 weights from hidden layer to output layer.

The identification system is done online, GA–PIDNN has many advantages such as simplicity, fast learning and powerful real-time performance, and this method is suitable for online learning and structure adjustment of multi-input systems. Therefore, this method can be applied in the problem of black box. Using GA–PIDNN nonlinear identification model of triple inverted pendulum as controlled object, the structure of GA–PIDNN controller is shown in Fig. 8.

5 The study of simulation

In order to verify the validity of the research method proposed in this paper, simulation experiments are conducted and the results are presented as follows.

5.1 Simulation study of identification system

According to the accurate, complete, nonlinear model of triple inverted pendulum advice, the input/output data can be obtained as the sample data of identification model.

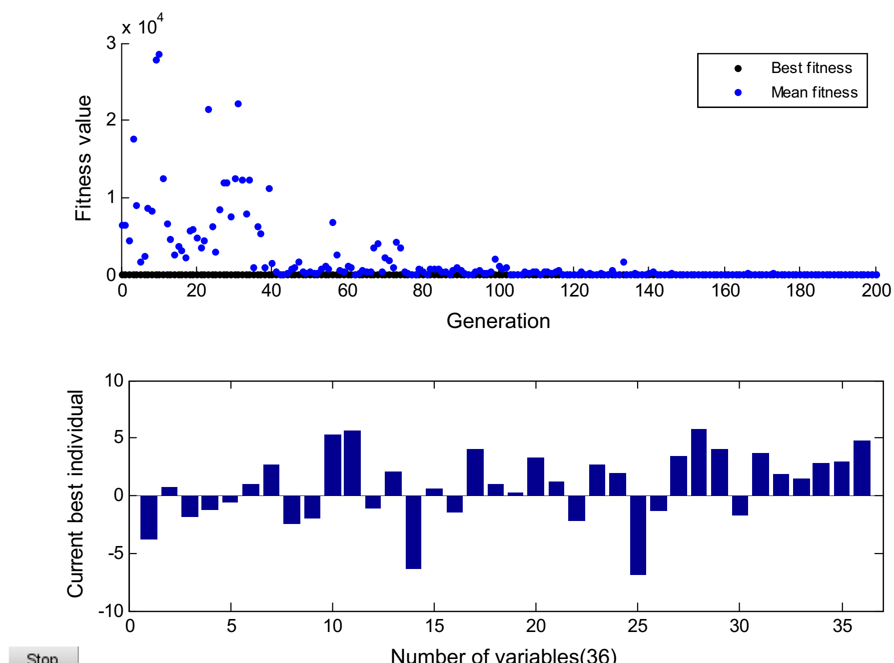

Fig. 11 The GA optimization results of control system by GA

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GA–PIDNN nonlinear identification model includes 72 weights to be optimized. The squared error is taken as the fitness function, and the curves of fitness function value and the best individual are shown in Fig. 9.

The simulation results of GA–PIDNN nonlinear identification system after online learning are shown in Fig. 10.

Figure 10 shows that GA–PIDNN identifier has a perfect tracking performance for the triple inverted pendulum. It can be observed that the identification has high precision, and the output curves of cart displacement and angel of three pendulums are basically coincident. With the same simulation conditions, the traditional BP algorithm is used to optimize PIDNN, and the comparative curves are shown in Fig. 10. We can see that the output curves of PIDNN based on GA is closer to the actual system than BP algorithm which verify the effectiveness of the designed method.

5.2 Simulation study of control system

GA–PIDNN controller includes 36 weights to be optimized. The curves of fitness function value and the best individual are shown in Fig. 11.

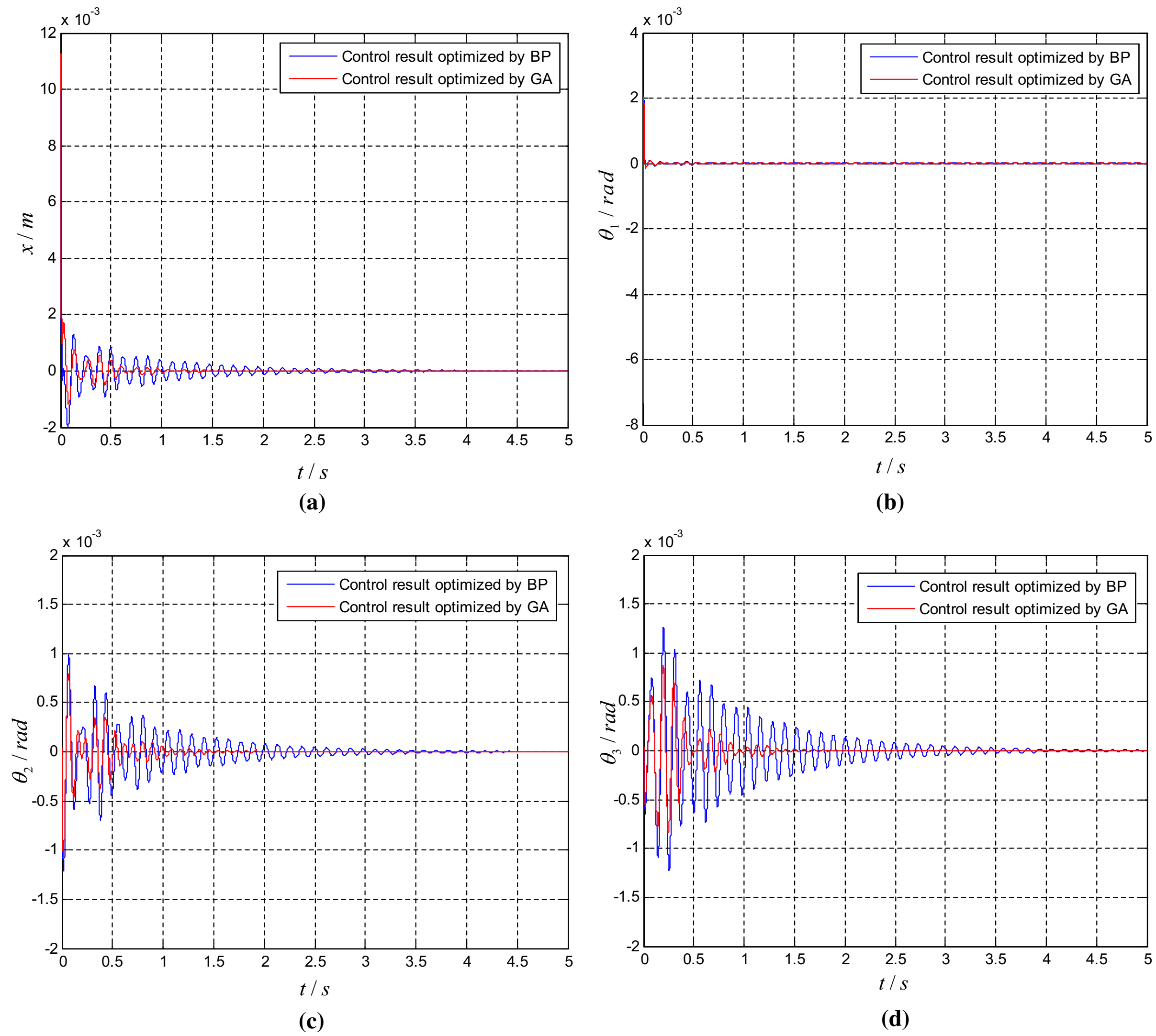

Fig. 12 The simulation results of control system. **a** The displacement of cart, **b** The angle of pendulum 1, **c** The angle of pendulum 2, **d** The angle of pendulum 3

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The simulation results of GA–PIDNN control system after online learning are shown in Fig. 12.

Fig. 12 shows that four controlled variables can achieve stability quickly by GA–PIDNN controller. Obviously, GA–PIDNN controller has the advantages of high accuracy, fast response and little overshoot over BP–PIDNN. GA has the character of global optimization which has not the shortage of easy trapped in local minimum. Therefore, it is a good control method which can meet the requirement of the triple inverted pendulum.

6 Conclusions

(1) PIDNN has the virtue of simple structure, the system easy to design and it need not require priori knowledge of the object system.
(2) Simulation results confirmed that GA could overcome intrinsic shortcomings of BP algorithm, including low learning efficiency, slow convergence rate and being easy to fall into local minima. Therefore, GA–PIDNN combined PIDNN with GA is a vivid method to solve the complex problem.
(3) Simulation results also demonstrate that GA–PIDNN nonlinear identification model of triple inverted pendulum has strong recognition ability and provides the basis for high-precision control.
(4) GA–PIDNN control system can achieve nonlinear control of triple inverted pendulum with simple structure. GA–PIDNN is an effective control method and provides new ideas for the control of nonlinear dynamic system.

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